

Minijets in γp and $\gamma\gamma$ collisions*

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Abstract

I discuss the minijet contribution to total photoproduction and photon–photon cross sections. While minijets with p_T around 2 GeV have recently been observed directly in $\gamma\gamma$ experiments, the total γp cross section measured at HERA is in excellent agreement with predictions based on purely soft physics. Due to the large number of free parameters, predictions for the minijet contribution to total cross sections can be brought into agreement with these seemingly paradoxical observations. However, the currently used eikonalization procedure may not be applicable at all to a large part of the minijet contribution, making it very difficult to draw definite conclusions at present.

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1) Introduction

The idea that “minijets”, i.e. partonic jets with $p_T \sim 1 - 3$ GeV, drive the observed increase of total hadronic cross sections with energy is now more than 20 years old [1]. While eikonalized minijet calculations can indeed describe this increase [2] it has to be admitted that a simple power-law formula, based on “old-fashioned” Pomeron physics, works just as well [3]. It had therefore been hoped that measurements of the total photoproduction cross section at HERA ($\sqrt{s_{\gamma p}} \simeq 200$ GeV) would serve to distinguish unambiguously between these two approaches.

This has unfortunately not happened. While the measurements by the H1 and ZEUS collaborations [4] fell just where estimates based on Pomeron exchange predicted [3] them to lie, proponents of the minijet idea were quick to point out [5, 6] that these results are not at all inconsistent with a large minijet contribution. Such “post-dictions” might have been deemed to be of dubious value if it were not for the fact that recently the TOPAZ and AMY collaborations unambiguously observed [7] minijets in $\gamma\gamma$ collisions at TRISTAN; see fig.1. These data, as well as other data on multi-hadron production in $\gamma\gamma$ scattering [8], imply that partonic collisions with transverse momenta in the GeV range occur at the rate predicted by perturbative QCD.

It has long been known [9] that the total *inclusive* cross section for the photoproduction of such minijets reaches the level of the total γp cross section just about at HERA energies. This raises the challenging question why this large inclusive cross section seems to make such a small contribution to the total cross section, leading to the observed quite modest increase of $\sigma_{\gamma p}^{\text{tot}}$ between 20 and 200 GeV. In order to tackle this problem, in sec. 2 I first describe current calculations of the minijet contribution to total cross sections, which are based on eikonalization. It quickly becomes apparent that these calculations contain sufficiently many free parameters or even unknown functions to accomodate just about any conceivable measurement of total cross sections, provided only they rise with energy. What is worse, these calculations might still be too simplistic; at least some of the underlying assumptions seem quite suspect to me, as explained in sec. 3. Finally, sec. 4 contains a brief summary and conclusions.

2) Eikonalized minijet cross sections

The calculation of inclusive jet cross sections is a straightforward application of perturbative QCD:

$$\sigma_{\gamma p}^{\text{jet}}(p_T > p_{T,\text{min}}) = \int_{x_{\text{min}}}^1 dx_1 \int_{x_{\text{min}}/x_1}^1 dx_2 f_{i|\gamma}(x_1) f_{j|p}(x_2) \int_{p_{T,\text{min}}}^{\sqrt{\hat{s}}/2} dp_T \frac{d\sigma_{ij}}{dp_T}, \quad (1)$$

with $x_{\text{min}} = 4p_{T,\text{min}}^2/s$ and $\hat{s} = x_1 x_2 s$. Here $f_{i|\gamma}$ and $f_{j|p}$ are distribution functions of partons i and j in the photon and proton, respectively, and $d\sigma_{ij}$ is the hard scattering cross section of these partons. The cross section (1) grows very quickly with energy. This can most easily be seen by assuming $f_{i|\gamma}, f_{j|p} \propto x^{-(J+1)}$ at small x ; using different powers for the parton

densities in the photon and proton does not change the result qualitatively [10]. Since $d\sigma_{ij}/dp_T \propto p_T^{-3}$ one has for $p_T^2 \ll s$:

$$\sigma_{\gamma p}^{\text{jet}}(p_T > p_{T,\text{min}}) \propto \frac{1}{J} \left(\frac{s}{4p_T^2} \right)^J \log \frac{s}{4p_T^2}. \quad (2)$$

The power J is expected to lie roughly in the range $0.2 \leq J \leq 0.5$. The r.h.s. of eq.(2) therefore grows much faster with energy than total cross sections do; experimentally, $\sigma^{\text{tot}} \propto s^{0.08}$ for both $\bar{p}p$ and γp collisions [3].

Recall, however, that eq.(1) describes an *inclusive* cross section. This differs from the minijet contribution to the total cross section by the average number of jet pairs (or partonic collisions) per hadronic collision:

$$\sigma_{\gamma p}^{\text{jet,tot}} = \sigma_{\gamma p}^{\text{jet}} / \langle n_{\text{jet}} \rangle. \quad (3)$$

In the usual eikonalization scheme [2, 5, 6, 10] the possibility of producing more than one jet pair in a γp collision is included by assuming that several parton-parton collisions occur *independently* of each other; the number of partonic collisions per event then obeys a Poisson distribution. In order to estimate the value of $\langle n_{\text{jet}} \rangle$ one also has to know the transverse overlap of the parton densities. In the usual treatment one makes the second crucial assumption that the dependence of the parton densities on Bjorken- x and on the impact parameter b factorizes.

Under these assumptions the total γp cross section can be computed from [10]:

$$\sigma_{\gamma p}^{\text{tot}} = P_{\text{had}} \int d^2b \left[1 - \exp \left(- \frac{\chi_{\text{soft}} + \sigma_{\gamma p}^{\text{jet}}(s)}{P_{\text{had}}} A(b) \right) \right]. \quad (4)$$

Here, $\sigma_{\gamma p}^{\text{jet}}$ is given by eq.(1). χ_{soft} is the “soft” (non-perturbative) contribution to the eikonal; it is mostly determined by low-energy data, but it has recently been argued [6] that it might show nontrivial s -dependence even at high energies. The function $A(b)$ describes the transverse overlap of the parton densities; it is normalized such that $\int d^2b A(b) = 1$.

Finally, the parameter P_{had} appearing in eq.(4) is supposed to describe the probability for a photon to go into a hadronic state. This is clearly $\mathcal{O}(\alpha_{em})$, but the exact value is not known. The necessity to introduce such a parameter has first been pointed out in ref.[10]. A very intuitive argument has been given in ref.[11]; it is based on the expansion of eq.(4) for small $\sigma_{\gamma p}^{\text{jet}}$. The n -th term in this expansion describes the cross section for the simultaneous production of n jet pairs. This gives:

$$\sigma(n \text{ jet pairs}) \propto P_{\text{had}} \left(\frac{\sigma_{\gamma p}^{\text{jet}}}{P_{\text{had}}} \right)^n. \quad (5)$$

Notice that $\sigma_{\gamma p}^{\text{jet}}$ in eq.(1) is $\mathcal{O}(\alpha_{em})$, since the $f_{i|\gamma}$ are $\mathcal{O}(\alpha_{em})$. If $P_{\text{had}} = 1$ the cross section for producing n jet pairs would therefore be $\mathcal{O}(\alpha_{em}^n)$. This is counter-intuitive; once the transition into a hadronic state has been made, no further electromagnetic interactions are needed to produce additional jet pairs. On the other hand, if $P_{\text{had}} \sim \mathcal{O}(\alpha_{em})$, eq.(5) gives $\sigma(n \text{ jet pairs}) \sim \mathcal{O}(\alpha_{em})$, as expected.

Clearly a great number of a priori unknown parameters and functions has to be fixed before eq.(4) can be evaluated. To begin with, the jet cross section $\sigma_{\gamma p}^{\text{jet}}$ depends on the parton densities, especially at small x and $Q^2 \sim p_{T,\text{min}}^2$. At least in principle these densities can be measured in processes that can be described by purely perturbative QCD. In contrast, $p_{T,\text{min}}$ is clearly not computable from perturbation theory alone; in fact, by introducing this parameter one hopes to describe the intricacies of confinement by a single parameter. A very similar cut-off parameter has been introduced in analyses [7, 8] of multi-hadron production in $\gamma\gamma$ reactions. Unfortunately different groups find different preferred values of $p_{T,\text{min}}$ even if they use the same set of structure functions. E.g., for the old DG parametrization [12], DELPHI finds a value as low as 1.45 GeV, while ALPEH data seem to favor $p_{T,\text{min}}$ around 2.5 GeV; results from AMY and TOPAZ lie in between. Fig.2 shows that such a variation of $p_{T,\text{min}}$ changes predictions for the minijet contribution to $\sigma_{\gamma p}^{\text{tot}}$ by at least a factor of 2 even at very high energies.

As discussed above, P_{had} has to be $\mathcal{O}(\alpha_{em})$ for eq.(4) to make sense at all; however, the exact value is unknown. The original estimate of ref.[10] was $P_{\text{had}} = 4\pi\alpha_{em}/f_\rho^2 \simeq 1/300$, but later a value of 1/170 has been suggested [11] based on parton model considerations. Fig. 3 shows that a factor-of-two uncertainty in P_{had} also leads to a substantial uncertainty in the prediction for $\sigma_{\gamma p}^{\text{tot}}$.

χ_{soft} is usually written in the form $A+B/\sqrt{s}$, making it independent of s at large energies. However, as already mentioned above, it has recently been suggested [6] that χ_{soft} might also grow slowly with energy. This will obviously affect predictions for $\sigma_{\gamma p}^{\text{tot}}$.

Finally, the function $A(b)$ needs to be specified. In all minijet calculations of $\sigma_{\gamma p}^{\text{tot}}$ that I am aware of $A(b)$ has been assumed to be the Fourier transform of the product of the electromagnetic form factors of the proton and of the pion. The use of electromagnetic form factors to estimate the transverse distribution of partons in the proton is certainly not unreasonable, although it does not allow for the occurrence of “hot spots”. Using π form factors as an estimate of the transverse parton distribution in the photon is quite a different matter, though. This approach is based on the VDM assumption that a photon is “basically” a vector meson (or a superposition of ρ , ω , ϕ and higher states), and the additional assumption that the ρ is “basically” like a π . To begin with, the ρ meson is really not much like a pion at all, being about 5 times heavier; indeed, since the ρ is a resonance, it can even be described as “consisting” of two pions! More seriously, we know experimentally that the x -dependence of the quark distribution functions in the photon does *not* look like that of the pion at $Q^2 \sim p_{T,\text{min}}^2 \sim (\text{a few}) \text{ GeV}^2$. In my view there is therefore no reason to assume that the b dependence is similar for the photon and the pion.

The difference in the x -dependence of photonic and pionic parton densities is largely due to the hard $\gamma q\bar{q}$ coupling. The existence of this pointlike vertex suggests to estimate the transverse distribution of partons in a photon as a Fourier transform of the hard intrinsic k_T distribution of the quarks produced in this vertex:

$$\begin{aligned} q^\gamma(b) &\propto \int d^2k_T \frac{\exp(-ik_T \vec{b})}{k_T^2 + k_0^2} \\ &\propto \int dk_T \frac{k_T}{k_T^2 + k_0^2} J_0(k_T b), \end{aligned} \tag{6}$$

where k_0 is an IR regulator and J_0 is a Bessel function. The distribution (6) is peaked at

$b = 0$; more importantly, its width is given by the inverse of the hard momentum scale in the problem, as opposed to the (rather large) radius of the pion. In other words, one expects this (hard) contribution to the parton densities in the photon to be more strongly peaked in transverse direction than in case of the pion. A narrower distribution means a larger $A(0)$, which increases eikonalization effects, i.e. reduces the predicted minijet contribution to $\sigma_{\gamma p}^{\text{tot}}$. A similar connection between $A(b)$ and the intrinsic k_T of the partons in the photon has been incorporated in the latest refinement [14] of the Schuler–Sjöstrand model [15] of photonic interactions; however, this model does not attempt to predict the total γp cross section (although it does predict the relative size of various contributions to that cross section).

3) Is eikonalization applicable at all?

The discussion at the end of the previous section raises doubts whether eikonalization should be used at all for those resolved photon contribution that come from the hard (perturbative) part of the photon structure functions. Recall that one of the fundamental assumptions in the derivation of eq.(4) was that multiple parton–parton reactions can occur *independently* in one γp scattering event. On the other hand, the entire perturbative part of the parton densities in the photon can by definition be traced back to the $\gamma q\bar{q}$ vertex. Given that all these partons manifestly originate from a common source it seems unlikely that they can be treated as being statistically independent. This is illustrated in fig. 4 for the case of two jet pairs. The sum of diagrams of the type shown on the left is supposed to be equal to the $\gamma \rightarrow$ hadrons transition probability multiplied with the square of the diagram to the right. Notice that it is assumed here that the first step, the $\gamma \rightarrow$ hadrons transition, can simply be described by a constant; in other words, the parton densities describing this “hadronic state” are assumed to have the same x and Q^2 dependence as the usual photonic parton densities, up to a constant factor. This is clearly a crude approximation at best.

Given that the applicability of eikonalization to a large part of resolved photon contributions is doubtful, it seems to be a good idea to look for experimentally measureable quantities that are sensitive to the existence of minijets but are *not* sensitive to eikonalization. Such quantities should therefore depend on the perturbatively calculable *inclusive* minijet cross section (1), rather than on its contribution to the total cross section.

Some time ago I suggested [16] that the product $\sigma^{\text{tot,inel}} \cdot \langle n_{\text{ch}} \rangle \cdot \langle p_{T,\text{ch}} \rangle$ might be a good candidate for such a quantity, where $\sigma^{\text{tot,inel}}$ is the total inelastic cross section, $\langle p_{T,\text{ch}} \rangle$ the average p_T of charged particles and $\langle n_{\text{ch}} \rangle$ the average charged particle multiplicity. The energy dependence of this quantity as measured in $\bar{p}p$ collisions is depicted in fig. 5; at least over the range shown here it seems to be described quite well by a simple linear function. This rapid increase is quite consistent with the rapid rise (2) of the inclusive minijet cross section. Notice that each of the three factors is predicted by minijet models to increase with energy. Indeed, one of the strengths of the minijet model is that it allows to estimate such quantities at all; its usefulness therefore goes well beyond the prediction of total cross sections. The prediction for each factor by itself depends on the eikonalization scheme, but the product should not depend on this: It should not be important whether two pairs of minijets are distributed over two γp events (giving large $\sigma^{\text{tot,inel}}$ but small $\langle p_{T,\text{ch}} \rangle \cdot \langle n_{\text{ch}} \rangle$) or are concentrated in one event (giving small $\sigma^{\text{tot,inel}}$ but large $\langle p_{T,\text{ch}} \rangle \cdot \langle n_{\text{ch}} \rangle$). Unfortunately,

this quantity is sensitive to fragmentation effects [17], since the scalar sum of the p_T of all (charged) particles does not add up to the p_T of the parton producing a minijet.

Nevertheless this (or a similar) quantity should be useful for determining the only non-perturbative parameter entering the calculation (1) of inclusive minijet rates, i.e. $p_{T,\min}$; here I assume that the relevant parton densities will be determined from other reactions (DIS, $c\bar{c}$ and J/ψ production, ...). The *inclusive* minijet cross section will then be known, and we can try to figure out how these minijets are distributed over γp events by studying details of these events, as done in ref.[18] for $\bar{p}p$ collisions. One possible problem of this approach is that the eikonalization ansatz (4) contains so many free parameters that it might be able to describe a large amount of data even if it is intrinsically flawed. Still, it seems clear to me that at present it is hopeless to try and make predictions for $\sigma_{\gamma p}^{\text{tot}}$ based on minijet models unless one uses either additional data or additional theoretical assumptions [5] as input; either way one has to go beyond the realm of perturbative QCD. Of course, the same remarks that I made here for γp scattering also apply for $\gamma\gamma$ reactions.

4) Summary and conclusions

Minijets exist (see fig. 1), but at present we are not able to compute their contribution to the total γp cross section reliably. As shown in sec. 2 the usual eikonalization prescription contains many unknown parameters. Even worse, in sec. 3 I have presented arguments casting doubt on the validity of this formalism for contributions coming from the perturbative part of photon structure functions. Unfortunately at present I cannot offer any alternative scheme to compute total cross sections from inclusive jet cross sections. It seems clear, though, that we have to use much more experimental information to determine, first, the *inclusive* minijet cross section at high energies, and in a next step, to figure out how these minijets are distributed over γp or $\gamma\gamma$ events. Only time will tell whether such a program will eventually force us to abandon conventional eikonalization schemes for γp and $\gamma\gamma$ reactions.

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Figure Captions

Fig.1 Example of reconstructed minijets observed by the TOPAZ collaboration [7].

Fig.2 Dependence of the minijet contribution to $\sigma_{\gamma p}^{\text{tot}}$ on $p_{T,\text{min}}$. The curves are for $p_{T,\text{min}} = 2, 3, 4, 5$ GeV. From ref.[13].

Fig.3 Dependence of the minijet prediction for $\sigma_{\gamma p}^{\text{tot}}$ on P_{had} ; from ref.[11].

Fig.4 In the usual eikonalization scheme, the diagram contributing to the simultaneous production of two minijet pairs (left) is supposed to be described by the square of the diagram shown on the right, multiplied with a constant $\gamma \rightarrow \text{hadrons}$ transition probability.

Fig.5 Energy dependence of $\sigma^{\text{tot,inel}} \cdot \langle n_{\text{ch}} \rangle \cdot \langle p_{T,\text{ch}} \rangle$ as measured at $\bar{p}p$ colliders; from ref.[16].

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